DETERMINATION OF ASSOCIATED SURFACES.

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IT is here proposed to develope a set of formulas for the cartesian coordinates of surfaces which are associate to a given surface referred to its asymptotic lines. The coordinates of an associated surface may thus be found directly from those of the given surface, by differentiation and substitution, when one has solved an equation of the Laplace type.

Since the images on the Gauss sphere of the asymptotic lines on any surface S_0 are the same as the images of a conjugate system of lines on the associated surface S, we may find S by determining the surface which has, as images of a conjugate system, the lines on the sphere which are the images of the asymptotic lines on S_0. Suppose S_0 is referred to its asymptotic lines, and let e, f, g represent the fundamental magnitudes of the corresponding sphere. Then in order that the lines $u =$ const., $v =$ const., on the sphere should be the images of the asymptotic lines on S_0 it is necessary and sufficient that

$$\frac{\partial}{\partial u}\begin{Bmatrix}1\,2\\1\end{Bmatrix}' = \frac{\partial}{\partial v}\begin{Bmatrix}1\,2\\2\end{Bmatrix}', \tag{1}$$

where the symbols of Christoffel are formed for the sphere. To find the coordinates of a surface S which has the above lines as images of a conjugate system, we proceed as follows: If X, Y, Z are the direction cosines of the normals to S along the conjugate lines, the cartesian coordinates x, y, z of S may be obtained by solving the equations *

$$\begin{cases} xX + yY + zZ = W, \\ x\dfrac{\partial X}{\partial u} + y\dfrac{\partial Y}{\partial u} + z\dfrac{\partial Z}{\partial u} = \dfrac{\partial W}{\partial u}, \\ x\dfrac{\partial X}{\partial v} + y\dfrac{\partial Y}{\partial v} + z\dfrac{\partial Z}{\partial v} = \dfrac{\partial W}{\partial v}, \end{cases} \tag{2}$$

where W is a solution, linearly independent of X, Y, Z, of the equation,

* Bianchi-Lukat, Vorlesungen über Differential-Geometrie, p. 139.