

A MODERN CALCULUS OF VARIATIONS.

Lectures on the Calculus of Variations. By OSKAR BOLZA. (The University of Chicago Decennial Publications, Second Series, Volume XIV.) The University of Chicago Press, Chicago, 1904. 8vo, xv + 271 pp.

THE calculus of variations is one of the very first of the old (or formal) developments of the infinitesimal calculus; it is one of the newest conquests of the modern (or critical) school. The history of the older calculus of variations is almost trite from reiteration; to select among many, the works of Moigno-Lindelöf, Diegner, Todhunter, Carll, Jordan, and (more recently and more perfectly) Pascal, have made known the achievements of the old school from Newton to Jacobi to mathematicians of all nations. The problems and the successes of the new school — the critical investigations — have been known to few until this century opened. In 1900 the first really modern treatise — Kneser's *Lehrbuch* — appeared. Without gainsaying the importance of that work and the fact that it opened the doors of modern research in the calculus of variations to the general mathematical public, it must be admitted that its style and arrangement are forbidding. The articles in the *Encyklopädie* (by Kneser and by Zermelo and Hahn) scarcely dispel the gloom of the *Lehrbuch's* unilluminated interior; but this was not to have been expected in encyclopedia articles, and the circumstances which necessitated two separate articles did not particularly conduce to public enlightenment on comparative values of various methods.

The book which is the subject of the present review is of interest to us because it is the first accurate and critical presentation of all the modern methods which is thoroughly readable, and because of the natural favorable prejudice of language and country. Apart from any such bias, however, Professor Bolza's treatise is surely admirable in many ways. Let us examine some of its notable features.

The preface is illuminating concerning much that has not been generally known, and the statements of the essential differences between the older theory and the new are worthy of recapitulation. They are: (1) the critical revision, by Weierstrass, Erdmann, Du Bois-Reymond, Scheefer, Schwarz, and others, of results previously obtained; (2) Weierstrass's intro-