Incidentally, the value of ρ obtained in (9) shows that the torsion, unlike the curvature, is independent of v.

An arbitrary field of force (1) *produces* ∞^5 *trajectories, of which* ∞^1 *pass through a given point in a given direction. These* ∞^1 *trajectories have, at the given point, a common osculating plane and a common torsion. The locus of centers of their osculating spheres is a straight line. Thus every field of force gives rise to a correspondence between the direction elements and the straight lines of space.*

COLUMBIA UNIVERSITY,
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ON THE POSSIBLE NUMBERS OF OPERATORS OF ORDER 2 IN A GROUP OF ORDER 2^m.

BY PROFESSOR G. A. MILLER.

(Read before the American Mathematical Society, September 7, 1905.)

It is well known that every group of order 2^m which contains only one operator of order 2 is either cyclic or it is composed of the cyclic group of order 2^{m-1} and 2^{m-1} operators of order 4 transforming each operator of this cyclic group into its inverse.* There are exactly two such groups for every value of $m > 2$. When $m = 3$ the latter of these two is the quaternion group, and when $m < 3$ the cyclic group is the only one that contains only one operator of order 2.

The groups of order 2^m in which the number of all the operators of order 2 is $\equiv 1$ mod. 4 have been determined incidentally in a recent paper.† Such groups exist only when the number of operators of order 2 is of the form $2^k + 1$, and there are exactly two possible groups for every arbitrary value of k. One of these is the dihedral rotation group of order 2^{k+1}, and the other is obtained by adding to the cyclic group of order 2^{k+1} an operator of order two which transforms each of its operators into its $(2^k - 1)$th power. Just half of the additional operators are of order two and the others are of order 4.

For instance, there are just two groups whose orders are of the form 2^m and which contain just five operators of order two;

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* Burnside, Theory of groups, 1897, p. 75.

† *Transactions Amer. Math. Society*, vol. 6 (1905), p. 62.