It now becomes necessary to show that the normal series E, E_1, $E_2, \cdots$; E, E_1, F_1, $F_2, \cdots$ define the same complementary series, and similarly for E, E'_1, $E'_2, \cdots$; E, E'_1, F_1, $F_2, \cdots$. This amounts to proving the theorem for the algebras E_1 and E'_1, which involves a finite number of repetitions of the above proof.

A chief or principal series E, P_1, $P_2, \cdots$ of an algebra being defined as one in which P_s is the maximal subalgebra of P_{s-1} which is invariant is $E(P_0 = E)$ it is easily shown in the symbolic notation, by exactly the same process as for the normal series, *that the system of complementary algebras* C_1, $C_2, \cdots$ *is independent of the chief series selected.*

In the case of the normal series the complementary algebras K_1, $K_2, \cdots$ are necessarily simple, but this is not true of the complementary algebras C_1, $C_2, \cdots$ in the case of the chief series.

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A GEOMETRIC PROPERTY OF THE TRAJECTORIES OF DYNAMICS.

BY DR. EDWARD KASNER.

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SUPPOSE that the force acting on a particle whose coordinates are x, y, z, produces an acceleration having the components $\phi\,(x, y, z)$, $\psi\,(x, y, z)$, $\chi\,(x, y, z)$. The equations of motion are then

$$(1)\qquad \ddot{x} = \phi\,(x, y, z),\ \ddot{y} = \psi\,(x, y, z),\ \ddot{z} = \chi\,(x, y, z),$$

where dots denote differentiation with respect to the time t. In such a field of force the initial position and initial velocity completely determine the trajectory. The totality of trajectories thus constitutes a quintuply* infinite system of space curves.

Consider now those trajectories obtained by starting the par-

* The only exception arises in the trivial case where the force is everywhere zero. Then the trajectories are the fourfold infinity of straight lines.