$$a = \Sigma x_1 i_1, \quad \beta = \Sigma y_1 i_1, \quad \gamma = \Sigma z_1 i_1,$$

$$a\beta\gamma = V_3 . \, a\beta\gamma + V_1 a\beta\gamma, \quad V_3 a\beta\gamma = \Sigma \mid x_1 y_2 z_3 \mid i_1 i_2 i_3,$$

$$V_1 a\beta\gamma = - \Sigma y_1 z_1 \Sigma x_1 i_1 + \Sigma x_1 z_1 \Sigma y_1 i_1 - \Sigma x_1 y_1 \Sigma z_1 i_1,$$

$$a\beta\gamma\delta = V_4 a\beta\gamma\delta + V_2 a\beta\gamma\delta + V_0 a\beta\gamma\delta,$$

and so on for higher products. The different " products " of the Ausdehnungslehre appear at once in their proper places as partial products among all the partial products of one *associative* product. This algebra is of order $2n$, and, if $n = 2m$, is the product of m independent quaternion algebras, if $n = 2m + 1$, is the product of m independent quaternion algebras and the algebra of positives and negatives. Division is possible under much the same restrictions as in a Weierstrass commutative algebra. We also see that the multiple algebra of n-dimensional space is not necessarily a theory of matrices of order n.

The volume is what its author intended it to be, a handy manual for those who desire to learn quaternions and quaternion methods. The appearance of the book is pleasing, and to the reviewer it seems that the simple notation of Hamilton can scarcely be called improved when one views a page full of heavy type, artificial signs, and foreign alphabets. We believe a letter is superior to an arbitrary mark for indicating a process, and indices clearer than fonts of type and sets of alphabets. Professor Joly's text speaks for itself. The book appeals, finally, to the pure mathematician as well as to the physicist.   JAMES BYRNIE SHAW.

SHORTER NOTICES.

Geschichte der Mathematik in XVI. und XVII. Jahrhundert. By H. G. ZEUTHEN, Deutsche Ausgabe von RAPHAEL MEYER. *Abhandlungen zur Geschichte der mathematischen Wissenschaften,* XVII. Heft. Leipzig, B. G. Teubner, 1903. Pp. viii + 434. Price 16 Marks.

AT first thought it may seem strange that Professor Zeuthen should attempt to go over the same ground so recently covered by Cantor in the latter part of volume II and the first part of volume III of the latter's Vorlesungen. A little investigation