A GEOMETRIC CONSTRUCTION FOR QUATERNION PRODUCTS.

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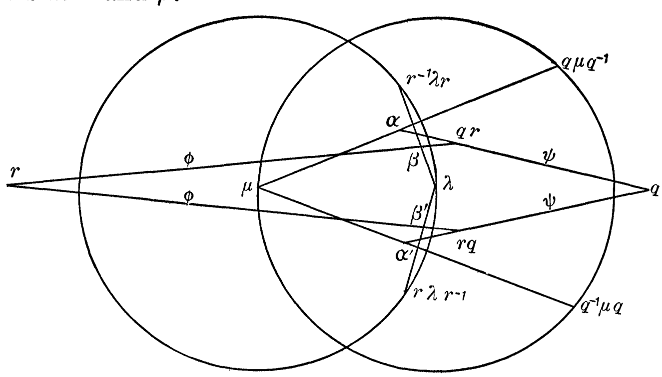
In the construction here given the lines actually inserted in the figure are supposed to lie on a hypersphere (spherical space of three dimensions) having a radius equal to the linear unit, and in particular all of the lines except the four marked ϕ, ψ, lie on a spherical surface passing through the extremities of the quaternion unit vectors i, j, k. The Greek letters that indicate the position of points are vectors drawn from the centre of the sphere to its surface and q, r, qr, rq are quaternion directors to points on the hypersphere. The centers of the sphere and hypersphere are coincident and are at the origin of vectors and of quaternion directors. The two circles of the figure have their centers at λ and μ.



The quaternions whose products are to be constructed are given in the form

$$q = \cos\phi + \lambda \sin\phi,$$
$$r = \cos\psi + \mu \sin\psi \qquad [Tq = Tr = 1].$$

It is well known that with a pair of quaternions q, r, there is associated a definite plane, whose equation is

$$\lambda t - tr^{-1}\lambda r = 0,^*$$

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