THE PRESENT PROBLEMS OF GEOMETRY.

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IN spite of the richness and power of recent geometry, it is noticeable that the geometer himself has become more modest. It was the ambition of Descartes and Leibniz to discover universal methods, applicable to all conceivable questions; later, the Ausdehnungslehre of Grassmann and the quaternion theory of Hamilton were believed by their devotees to be ultimate geometric analyses; and Chasles attributed to the principles of duality and homography the same rôle in the domain of pure space as that of the law of gravitation in celestial mechanics. To-day the mathematician admits the existence and the necessity of many theories, many geometries, each appealing to certain interests, each to be developed by the most appropriate methods; and he realizes that, no matter how large his conceptions and how powerful his methods, they will be replaced before long by others larger and more powerful.

Aside from the conceivability of other spaces with just as self-consistent properties as those of the so-called ordinary space, such diverse theories arise, in the first place, on account of the variety of objects demanding consideration — curves, surfaces, congruences and complexes, correspondences, fields of differential elements, and so on in endless profusion. The totality of configurations is indeed not thinkable in the sense of an ordinary assemblage, since the totality itself would have to be admitted as a configuration, that is, an element of the assemblage.

However, more essential in most respects than the diversity in the material treated, is the diversity in the points of view from which it may be regarded. Even the simplest figure, a triangle or a circle, has an infinity of properties — indeed, recalling the unity of the physical world, the complete study of a single figure would involve its relations to all other figures and thus not be distinguishable from the whole of geometry. For the past three decades the ruling thought in