ON THE DEFORMATION OF SURFACES OF TRANSLATION.

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It is here proposed to find all surfaces of translation which may be deformed so that their generating lines remain generating lines throughout the deformation. The generating lines, $u =$ const., $v =$ const., of a surface of translation

$$(1) \quad x = f_1(u) + \phi_1(v), \quad y = f_2(u) + \phi_2(v), \quad z = f_3(u) + \phi_3(v)$$

form a conjugate system, since x, y, z satisfy an equation of the form *

$$(2) \qquad \frac{\partial^2 \theta}{\partial u \partial v} = a \frac{\partial \theta}{\partial u} + b \frac{\partial \theta}{\partial v}.$$

The square of the lineal element of (1) is,

$$(3) \qquad ds^2 = E du^2 + 2F du dv + G dv^2,$$

where

$$E = \Sigma f_i'^2, \quad F = \Sigma f_i' \phi_i', \quad G = \Sigma \phi_i'^2 \qquad (i = 1, 2, 3)$$

and the primes denote derivatives of u and v respectively.

Forming the symbols of Christoffel from (3) we have

$$(4) \qquad \begin{Bmatrix} 1 & 2 \\ & 1 \end{Bmatrix} = 0 \quad \text{and} \quad \begin{Bmatrix} 1 & 2 \\ & 2 \end{Bmatrix} = 0.$$

Conversely, if a surface S is referred to a conjugate system of lines and if (4) is true, then S must be a surface of translation, since its coördinates x, y, z must satisfy

$$\frac{\partial^2 \theta}{\partial u \partial v} = \begin{Bmatrix} 1 & 2 \\ & 1 \end{Bmatrix} \frac{\partial \theta}{\partial u} + \begin{Bmatrix} 1 & 2 \\ & 2 \end{Bmatrix} \frac{\partial \theta}{\partial v},$$

the integration of which gives (1) by virtue of (4). Hence we have the following

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* Bianchi-Lukat: Vorlesungen über Differentialgeometrie, p. 110.