Hence our differential equation becomes

$$\frac{c}{\sqrt{Y^2-c^2}}=\frac{Y^2}{Y'},$$

the integral of which is

$$\frac{\sqrt{Y^2 - c^2}}{Yc} = X + \text{constant.}$$

If we substitute the old variables, this becomes

$$(x^{2} + y^{2})(k^{2}c^{2} - 1) + 2c^{2}kx + c^{2} = 0,$$

where k is an arbitrary constant, and this is the required general integral.

Again, we know that if f(x, y, p) = 0 admits the group  $\xi f_x + \eta f_y$ ,  $f(x, y, \eta/\xi) = 0$  satisfies the equation, and contains as factor any singular solution of the equation. If there are any other factors, they are particular cases of the general integral for certain values of the arbitrary constant.

In our case  $f(x, y, \eta/\xi) = 0$  becomes

$$\left[(x^2+y^2)^2-c^2y^2\right](x^2+y^2)=0.$$

Here  $x^2 + y^2 = 0$  is the general integral when  $k = \infty$ , and

$$(x^2 + y^2)^2 - c^2 y^2 = 0$$

is the singular solution.

BRYN MAWE, PA., October, 1904.

## ON THE QUINTIC SCROLL HAVING A TACNODAL OR OSCNODAL CONIC.

## BY PROFESSOR VIRGIL SNYDER.

## (Read before the American Mathematical Society, October 29, 1904.)

BESIDES the quintic scrolls having three double conics which were discussed in the BULLETIN (volume 9, pages 236–242), other particular types exist. Two of the double conics may become consecutive, forming a tacnodal conic; or all three may become consecutive, forming an oscnodal conic. The necessary

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