ON DOUBLY INFINITE SYSTEMS OF DIRECTLY SIMILAR CONVEX ARCHES WITH COMMON BASE LINE.

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IN his paper "The determination of the constants in the problem of the brachistochrone" (BULLETIN, January, 1904, pages 185–188) Professor Bolza has proved analytically the statement of Weierstrass (lectures of 1882) that, of all cycloid arches having bases in a given line and lying on one side of that line, precisely one passes through any two points, lying on that side of the line or one or both lying on the line, and not lying in the same perpendicular to the line.

The special case in which one of the points is on the base line was handled geometrically by the brothers Bernoulli (cf. Ostwald's "Klassiker der exacten Wissenschaften," number 46, pages 12 and 18), use being made of the fact that all cycloid arches are similar. In reading his paper before the mathematical club of the University of Chicago, November 20, 1903, Professor Bolza, after indicating this solution, added a geometric solution for the case in which the segment is parallel to the base line, use being made furthermore of the fact that a cycloid arch is symmetric with respect to the perpendicular bisector of its base.

In the present note I give an analytically phrased geometric proof of the statement of Weierstrass in its generality, at the same time extending it (cf. theorem I_0, §3) to cover the case of *any doubly infinite system of directly* * *similar convex arches possessing tangents and meeting perpendicularly a common base line.*

One obtains an obvious generalization of this theorem in replacing perpendicularity to the base line by parallelism to any given line not parallel to the base line. More essential generalizations remain for consideration. In a subsequent note I propose to take up the connections of this theorem with the calculus of variations.

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* The distinction between direct and inverse similarity enters only in case the arches do not have the symmetry spoken of above.