$$+ \cdots + \alpha'^{\phi(P)} \pi^{\phi(P)} \equiv 0, \text{ mod. } P^{n+1},$$

where $\alpha\alpha' \equiv 1$, mod. P^n.

If $m < n$, all the terms in (8) would be divisible by P^m, and hence $\phi(P)$ divisible by P, which is impossible. Hence we must have $m = n$. Then we get from (8)

$$\pi \equiv \alpha\mu_n q_n(\alpha), \text{ mod. } P^{n+1},$$

and

$$(9) \qquad \beta \equiv \alpha[1 + \mu_n q_n(\alpha)], \text{ mod. } P^{n+1}.$$

It is also easily seen that $\alpha[1 + \mu_n q_n(\alpha)]$ is a root of (7), if α is a root of (6). Now let α_1 and α_2 be two roots of (6), incongruent mod. P^n. Then, if

$$\alpha_1[1 + \mu_n q_n(\alpha_1)] \equiv \alpha_2[1 + \mu_n q_n(\alpha_2)], \text{ mod. } P^{n+1},$$

we should have

$$\alpha_1 - \alpha_2 \equiv \mu_n[\alpha_2 q_n(\alpha_2) - \alpha_1 \mu_n q_n(\alpha_1)], \text{ mod. } P^{n+1},$$

which is impossible, since $\alpha_1 - \alpha_2$ is not divisible by P^n.

Now by giving to n the values 1, 2, 3, $\cdots$ we thus see that all the roots of

$$x^{\phi(P)} \equiv 1, \text{ mod. } P^n,$$

are

$$(10) \quad x \equiv \alpha[1 + \mu_1 q_1(\alpha)] \cdots [1 + \mu_{n-1} q_{n-1}(\alpha)], \text{ mod. } P^n,$$

where α runs through the roots of

$$x^{\phi(P)} \equiv 1, \text{ mod. } P.$$

PURDUE UNIVERSITY,
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MACH'S MECHANICS.

The Science of Mechanics — a Critical and Historical Account of its Development. By ERNST MACH. Translated from the German by T. J. McCORMACK. Second revised and enlarged edition. Chicago, The Open Court Publishing Co., 1902. xix + 605 pp.

IN a recent review of the German edition of Routh's Rigid Dynamics, BULLETIN, May, 1902, we expressed the desire