A MODERN FRENCH CALCULUS.

Cours d'Analyse Mathématique. Par ÉDOUARD GOURSAT, Professeur à la Faculté des Sciences de Paris. Tome I. Gauthier-Villars, Paris, 1902.

THE revision of the fundamental principles of the calculus, which was initiated by Cauchy and Abel and carried through by Weierstrass and his followers, led to the development of the ϵ-proof (early introduced by Cauchy) and to the precise formulation of definitions and theorems. In Germany and Italy a tendency sprang up to place only such restrictions on definitions and theorems as are necessarily imposed by the nature of the case. Thus functions continuous throughout no interval whatever were admitted as the integrand of a definite integral simply because the form of the definition of the integral applied to a certain class of these functions, and the question was examined of how far the ordinary theorems of the integral calculus hold for such functions. Again, the theorem that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ was proved with fewer restrictions than the continuity of all the derivatives that enter. While this procedure is perfectly justifiable so far as it is a question of research in a special field, it is important not to lose sight of the fact that investigations of this sort are but a very special phase of modern analysis, and that even the specialist in the field of analysis may never need to trouble himself about the integrals of other functions than those which are continuous except at a finite number of points. That which is essential for every mathematician to know who has occasion to use the calculus to any extent is a simple formulation of the theorems and simple tests for the validity of the processes of the calculus which have been handed down to us from Euler's time and earlier: — when may a convergent series of continuous functions be integrated term by term, when may a definite integral whose integrand satisfies reasonable conditions of continuity be differentiated under the sign of integration? These are questions of general interest to mathematicians. To the importance of a simple and lucid answer French mathematicians are alive. With full appreciation of modern standards of rigor they do