THE THEORY OF AUTOMORPHIC FUNCTIONS.

Vorlesungen über die Theorie der Automorphen Functionen.
Von ROBERT FRICKE und FELIX KLEIN. Leipzig, B. G.
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SIDE by side with the growth of general function theory special classes of functions have developed in which the general theories have found abundant opportunity to display their fertility and power. On the other hand, the study of special functions has repeatedly afforded the stimulus and suggested the path for new investigations along general lines. When, in addition, the intrinsic value and usefulness of such functions as the elliptic, hyperelliptic, abelian, hypergeometric, Bessel, etc., is taken into consideration, it is readily seen that the cultivation of such special fields is scarcely second in interest and importance to that of the general theory itself.

Among the various classes of special functions which have hitherto engaged the attention of mathematicians, that of most recent origin, and of by far the largest content (at least potentially) is the automorphic functions. This vast subject, the growth of the past quarter of a century, owes its rapid development to the genius and assiduity of the two eminent mathematicians, Klein and Poincaré, as well as to the comparatively high state of perfection of other mathematical disciplines which have been forced to contribute their assistance to this new field. Klein and Poincaré have each brought to this subject a breadth of knowledge and a corresponding wealth of ideas truly remarkable at the present day when multiplicity of interests hardly permits the investigator any other choice than to specialize within limits more or less narrow.

Klein in particular has by precept and example urged the importance of a closer unity among all departments of mathematical thought, and the great advantage to be derived from bringing to the aid of any one field the combined resources of all the others. It is in his work on the automorphic functions that he has given the most brilliant illustration of this mode of