separation of science into constituent parts, while there is ultimately a branching into the many distinct sciences. The troublesome problem of the closer relation of pure mathematics to its applications : can it not be solved by indirection, in that through the whole course of elementary mathematics, including the introduction to the calculus, there be recognized in the organization of the curriculum no distinction between the various branches of pure mathematics and likewise no distinction between pure mathematics and its principal applications? Further, from the standpoint of pure mathematics : will not the twentieth century find it possible to give to young students during their impressionable years in thoroughly concrete and captivating form the wonderful new notions of the seventeenth century? By way of suggestion these questions have been answered in the affirmative, on condition that there be established a thoroughgoing laboratory system of instruction in primary schools, secondary schools, and junior colleges — a laboratory system involving a synthesis and development of the best pedagogic methods at present in use in mathematics and the physical sciences.

CONCERNING THE AXIOM OF INFINITY AND MATHEMATICAL INDUCTION.

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I. *Introductory Considerations.*

This paper deals with a question which, on the one hand, is a question of pure logic, and, on the other, a question of Mengenlehre. It is often asserted, and is probably true, that reasoning naturally takes place in accordance with what the logicians of the school called first intentions. But ratiocination as activity, however unconscious its conformation to law, is nevertheless not lawless; and from the period when this fact came clearly into the consciousness of the Greek mind, as early as the time of Protagoras,* science has been neither able nor

* The so-called laws of thought seem to have struggled into consciousness mainly through the disputations of the Sophists. The law of contradiction, in particular, appears to have received its earliest formulation in the καταβάλλοντες of Protagoras. Cf. Windelband : Geschichte der Philosophie, and Ueberweg : System der Logik (both works also in English).