Since $S_1 T = c^{-1}b^{-1}cbc$ is the transform of c by bc, it is of period three.

The final relation (10) becomes

$$(bc^{-1}b^{-1}c \cdot b^{-1}cbc)^2 = (c^{-1}bcb^{-1} \cdot b^{-1}cbc)^2 = (c^{-1}bcb^2cbc)^2$$
$$= c^{-1}b(cb^2)^4 b^{-1}c = I.$$

Since S_j is commutative with S_1, the condition $S_j^3 = I$ follows from $(b^{-1}c^{-1}b^2c^{-1})^3 = I$ or $(cb^2cb)^3 = I$.

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NOTE ON A PROPERTY OF THE CONIC SECTIONS.

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IT is easily proved that if P, Q, R are any three points on the conic $Ax^2 + By^2 = 1$, and O the center of the conic, then the areas of the triangles OPQ, OPR, OQR will satisfy an equation independent of the position of the points P, Q, R. If a, b, c are the areas in question, this equation is

$$(1) \quad a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 - 2b^2c^2 + 16ABa^2b^2c^2 = 0.$$

Now we can prove that such an invariant relation is possible for no plane curves except the central conics; *i. e., if we seek a plane curve C and a point O in its plane such that, if P, Q, R are any three points on C, the triangles OQR, ORP, OPQ are connected by a relation independent of the coördinates of the points P, Q, R, we find C to be a central conic section and O its center.*

To prove this theorem, let O be the origin of coördinates, and let the coördinates of P, Q, R be respectively x_1, y_1; x_2, y_2; x_3, y_3. Then twice the areas of the three triangles are

$$2a = \pm (y_2x_3 - y_3x_2), \quad 2b = \pm (y_3x_1 - y_1x_3),$$
$$2c = \pm (y_1x_2 - y_2x_1),$$