INFINITESIMAL DEFORMATION OF THE SKEW HELICOID.

BY DR. L. P. EISENHART.

(Read before the American Mathematical Society, September 3, 1902.)

CONSIDER the skew helicoid S, defined by the equations

$$x = u \cos v, \quad y = u \sin v, \quad z = av. \tag{1}$$

We shall show that the problem of the infinitesimal deformation of this surface can be completely solved.

By direct calculation we find

$$\begin{aligned} E = \sum \left(\frac{\partial x}{\partial u} \right)^2 = 1, \quad F = \sum \frac{\partial x}{\partial u} \frac{\partial x}{\partial v} = 0, \\ G = \sum \left(\frac{\partial x}{\partial v} \right)^2 = u^2 + a^2, \end{aligned} \tag{2}$$

and

$$X, Y, Z = \frac{a \sin v, \; - a \cos v, \; u}{\sqrt{u^2 + a^2}} \tag{3}$$

where Y, X, Z denote the direction cosines of the normal. Again we find

$$\begin{aligned} D = \sum X \frac{\partial^2 x}{\partial u^2} = 0, \quad D' = \sum X \frac{\partial^2 x}{\partial u \, \partial v} = \frac{-a}{\sqrt{u^2 + a^2}}, \\ D'' = \sum X \frac{\partial^2 x}{\partial v^2} = 0. \end{aligned} \tag{4}$$

The characteristic equation of the deformation reduces in this case to

$$\frac{\partial^2 \phi}{\partial u \partial v} + \frac{u}{u^2 + a^2} \frac{\partial \phi}{\partial v} = 0,$$

of which the general integral is

$$\phi = \frac{U + V}{\sqrt{u^2 + a^2}}, \tag{5}$$