CONCERNING THE COMMUTATOR SUBGROUPS OF GROUPS WHOSE ORDERS ARE POWERS OF PRIMES.

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IN the *Transactions* of the AMERICAN MATHEMATICAL SOCIETY, volume 3 (1902), pages 331, 351, the writer has shown that the commutators of a metabelian group are invariant in the group, and that those operators of a group which correspond to the invariant operators of the group of cogredient isomorphisms are commutative with all the commutators of the group. It follows as a direct consequence of these two facts that the commutator subgroup of a group of the second or third class is abelian, that is, is of the first class. It was shown in the same article, page 349, that a metabelian group of odd order cannot be a group of cogredient isomorphisms if it has a set of generators such that the order of any one of them is not a divisor of the least common multiple of all the others. It is the main object of this paper to show that these facts are special cases of more general ones.

Let G be a group of order p^m (p being a prime) and let G' be the group of cogredient isomorphisms of G, and G'' that of G'; also let l and l'' denote the classes of the commutator subgroups of G and G'' respectively. If B_1 is any commutator of G, and A_i $(i = 1, 2, \cdots, l'')$ a set of any l'' commutators (not necessarily distinct) of G, we have $A_i^{-1}B_iA_i = B_iB_{i+1}$. Now since the commutator subgroup of G'' is of class l'' it is evident that $B_{l''}''$ is invariant in this subgroup ($B_{l''}''$ being that operator of G'' that corresponds to $B_{l''}$). Therefore $B_{l''+1}$ is invariant in the commutator subgroup of G and $l \leqq l'' + 1$. We have seen that for groups of classes two or three the commutator subgroups are of class one. We have therefore proved the

THEOREM: *If a group G is of order p^m (p being a prime) and class $2k$ or $2k+1$, its commutator subgroup is of class l, where $l \leqq k$.*

Suppose now that G is of class k, where $k \leqq p$, and has an abelian commutator subgroup. Let A_i' $(i = 1, 2, \cdots, n)$, be a set of generators of G' of orders p^{a_i} respectively, and let A_i be