of referring the plane to lines or lineoids as coördinate elements, to determine the plane by reference to planes, as follows: *if from the plane assemblage in question six planes* $\pi_1, \cdots, \pi_6$ *be chosen such that they furnish three pairs* $\pi_1\pi_6$, $\pi_2\pi_5$, $\pi_3\pi_4$ *of absolutely perpendicular planes, then we may take for homogeneous plane coördinates of a plane* π *the six cosine products* $\cos\theta_a \cos\omega_a$, *where* θ_a *and* ω_a *are the angles of* π *with the coördinate plane* π_a.

The six coördinate planes intersect an arbitrary lineoid of 4-space in the six edges of a tetraedron, while the arbitrary plane π cuts from the lineoid an equally arbitrary line. The angles θ and ω correspond to the distances and angles of the line with the tetraedral lines. The corresponding interpretation of the Plücker coördinates would accordingly be one in terms of these distances and angles.

We will close this note with the necessary and sufficient condition that two planes (θ_a, ω_a) and (θ_a', ω_a') shall have a common line. It is

$$\begin{aligned} &\cos\theta_1 \cos\theta_6' \cos\omega_1 \cos\omega_6' + \cos\theta_1' \cos\theta_6 \cos\omega_1' \cos\omega_6 \\ (19)\quad &- \cos\theta_2 \cos\theta_5' \cos\omega_2 \cos\omega_5' - \cos\theta_2' \cos\theta_5 \cos\omega_2' \cos\omega_5 \\ &+ \cos\theta_3 \cos\theta_4' \cos\omega_3 \cos\omega_4 + \cos\theta_3' \cos\theta_4 \cos\omega_3' \cos\omega_4 = 0. \end{aligned}$$

COLUMBIA UNIVERSITY,
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NOTE ON THE SUFFICIENT CONDITIONS FOR AN ANALYTIC FUNCTION.

BY MR. D. R. CURTISS.

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SINCE the publication of Goursat's proof* that a function of the complex variable z possessing a derivative at each point of a two-dimensional region T in which it is single-valued must necessarily have a continuous derivative throughout that region, the question has arisen whether the sufficient conditions for an analytic function, stated in terms of the partial derivatives of the real and pure imaginary parts of the function, may not be reduced to simpler terms. These conditions are ordinarily given as follows:

$w = u(x, y) + iv(x, y)$ is an analytic function of the complex variable $z = x + iy$ at each point of a region T of the z-plane if throughout T

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* *Trans. Am. Math. Soc.*, Vol. 1 (1900), No. 1, p. 14.