NOTE ON THE TRANSFORMATION OF A GROUP INTO ITS CANONICAL FORM.

BY DR. S. E. SLOCUM.

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LET

$$X_j \equiv \sum_1^n {}_k \xi_{jk}(x_1, \cdots, x_n) \frac{\partial}{\partial x_k} \quad (j = 1, 2, \cdots, r),$$

where the ξ's are analytic functions of n independent variables $x_1, \cdots, x_n$), denote r independent infinitesimal transformations of a given r parameter group. The finite equations of the one-parameter groups generated by each of the infinitesimal transformations X_j ($j = 1, 2, \cdots, r$) may be obtained by integration of the r simultaneous systems

$$\frac{dx_1'}{\xi_{j1}(x_1', \cdots, x_n')} = \cdots = \frac{dx_n'}{\xi_{jn}(x_1', \cdots, x_n')} = da$$
$$(j = 1, 2, \cdots, r),$$

subject to the condition that $x_i' = x_i$ ($i = 1, 2, \cdots, n$) for $a = 0$, a being an arbitrary parameter. Let the integrals of these simultananeous systems be represented by the equations

$$x_i' = f_{ik}(x_1, \cdots, x_n, a) \quad (i = 1, 2, \cdots, n\,;\; k = 1, 2, \cdots, r).$$

Performing upon the manifold $x_1, \cdots, x_n$ a general transformation T_1 of the one parameter group generated by X_1 we obtain the manifold $x_1', \cdots, x_n'$; performing upon this latter manifold a general transformation T_2 generated by X_2, we obtain the manifold $x_1'', \cdots, x_n''$, etc. Thus we have

$$x_1' = f_{11}(x_1, \cdots, x_n, a_1), \quad \cdots x_n' = f_{n1}(x_1, \cdots, x_n, a_1),$$
$$x_1'' = f_{12}(x_1', \cdots, x_n', a_2), \quad \cdots x_n'' = f_{n2}(x_1', \cdots, x_n', a_2),$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$x_1^{(r)} = f_{1r}(x_1^{(r-1} \cdots, x_n^{(r-1)}, a_r), \cdots x_n^{(r)} = f_{nr}(x_1^{(r-1)}, \cdots, x_n^{(r-1)}, a_r),$$

where $a_1, \cdots, a_r$ are arbitrary parameters. Eliminating $x_1', \cdots, x_n^{(r-1)}$ between these equations, we have

$$x_i^{(r)} = f_i(x_1, \cdots, x_n, a_1, \cdots, a_r) \quad (i = 1, 2, \cdots, n),$$