where ρ_1 and ρ_2 denote the principal radii of S. For the second of the conditions (5) this gives $\rho_1 = \rho_2$, that is, S is a sphere, when real. Combining these results we have that *minimal surfaces and spheres are the only real surfaces for which the spherical representation of lines of length zero is the system of rectilinear generatrices of the sphere.*

In order to determine what surfaces have the lines of length zero on the sphere for spherical representation of its asymptotic lines we note that for the parameters used previously the equation of the asymptotic directions is*

$$rdu^2 + 2(s + z)dudv + tdv^2 = 0.$$

Hence, in order that the u and v lines may be asymptotic for the surface, the condition is

$$r = t = 0,$$

which as we have seen characterizes the sphere. Again, in order that the parametric lines may form a conjugate system we get from the above equation the condition

$$s + z = 0,$$

that is minimal surfaces. Recalling the above theorem we have the following:

In order that the asymptotic lines or a conjugate system on a surface may be represented upon the sphere by its imaginary generatrices, they must be lines of length zero on the surface.

PRINCETON,
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SOME PROPERTIES OF POTENTIAL SURFACES.

BY DR. EDWARD KASNER.

(Read before the American Mathematical Society, April 27, 1901.)

IN a previous paper, published in the BULLETIN, volume 7, pages 392–399, the author studied the algebraic curves $\varphi(x\,y) = 0$ defined by the condition $\varphi_{xx} + \varphi_{yy} \equiv 0$. Two classes of characteristic properties were obtained, the first translating directly the differential equation and the second arising from the well-known connection between harmonic

* Darboux, Leçons, I, p. 246.