just seen, its coefficients are rational. It follows from the definition that the equation (1) is reducible.

Conversely, it is now easy to see that if the equation (1) is reducible, its group must be intransitive.

We are therefore enabled to enunciate the following theorem, exactly analogous to a theorem in algebra :

The group of an irreducible linear differential equation is transitive.

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LINES OF LENGTH ZERO ON SURFACES.

BY DR. L. P. EISENHART.

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BY definition, the directions of the double system of lines of length zero on a surface are given by equating to zero the expression for the square of the linear element of the given surface, that is, in the Gauss notation,

$$(1) \qquad Edu^2 + 2Fdudv + Gdv^2 = 0.$$

Since these lines are always imaginary and since the lines of curvature of a real surface are always real there is no real surface whose lines of curvature are of length zero. It is well known that lines of length zero form a conjugate system upon a minimal surface and further that this is a characteristic property of such surfaces. In order that asymptotic lines be of length zero it is necessary and sufficient that the two fundamental forms of the surface be proportional. Hence the sphere is the only real surface whose asymptotic lines are of length zero ; they are the imaginary rectilinear generatrices of the sphere. When the parameters of these lines are taken as conjugate imaginaries and the sphere is of radius unity and with center at the origin, the cartesian coördinates have the following expressions :

$$(2) \qquad x = \frac{u+v}{1+uv}, \quad y = i\,\frac{v-u}{1+uv}, \quad z = \frac{uv-1}{1+uv}.*$$

We propose now the following problem :

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* Darboux, Leçons, I, p. 245.