every group of the fourth class, of order p^5, where $p > 3$, that has an operator of order p^2 is generated by an operator of order p^2 and three of order p.* No such group can be a group of cogredient isomorphisms. Therefore:

A group of order p^m, where p is a prime > 3, that contains an operator of order p^{m-3} is of class k, where $k \gtreqless 4$.

CORNELL UNIVERSITY.

PROOF THAT THE GROUP OF AN IRREDUCIBLE LINEAR DIFFERENTIAL EQUATION IS TRANSITIVE.

BY DR. SAUL EPSTEEN.

(Read before the American Mathematical Society, December 28, 1901.)

WE will define with Frobenius † the linear differential equation

$$(1) \qquad \frac{d^n y}{dx^n} + p_1 \frac{d^{n-1} y}{dx^{n-1}} + \cdots + p_n y = 0$$

to be irreducible when it has no integral in common with a differential equation of the same character, but of a lower order.

Picard has shown ‡ that for the equation (1) a linear homogeneous group

$$(2) \qquad Y_i = \sum_k a_{ik} y_k$$

plays the same rôle as the group of substitutions plays in the Galois theory of algebraic equations.

It is the object of this brief paper to show that the group (2) of the equation (1) will be transitive § when the equation is irreducible and intransitive when the equation is reducible, and we shall carry the proof out on lines analogous to the corresponding theorem in algebra. ||

The basis of our proof is the Lagrange-Vessiot theorem, ¶ that if the rational differential function $S(y)$ of the in-

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* Bagnera, *Ann. di Matematica*, ser. 3, vol. 1 (1898), p. 218.

† Frobenius, *Crelle*, vol. 76.

‡ Picard, *Comptes rendus*, 1883.

§ Lie-Engel, Transformationsgruppen, I., ch. 13.

|| Netto, Substitutionstheorie, § 154.

¶ E. Vessiot, *Annales de l'Ec. Norm. Sup.*, 1892.