$$x_{\alpha\beta} = \frac{x_\alpha + x_\beta}{2} + i\frac{y_\alpha - y_\beta}{2}, \quad y_{\alpha\beta} = \frac{y_\alpha + y_\beta}{2} - i\frac{x_\alpha - x_\beta}{2}.$$

As is well known, the n quantities

$$R(z_\alpha) \qquad (\alpha = 1, 2, \cdots, n),$$

where R denotes any rational function, satisfy an equation of the nth degree

$$F_n(t) = 0,$$

whose coefficients are rational in $A_0, A_1, \cdots, A_n$. This, however, no longer holds when we consider, instead of rational functions of the roots, rational functions of the real and imaginary parts of the roots; but if we consider the n^2 quantities

$$R(x_{\alpha\beta}, y_{\alpha\beta}),$$

they will satisfy an equation of the n^2 degree with coefficients which are rational in terms of the coefficients of φ and ψ, *i. e.*, in terms of $b_0, \cdots, b_n, c_0, \cdots, c_n$. Therefore,

THEOREM XV. *The n quantities*

$$R(x_1, y_1), \quad R(x_2, y_2), \cdots, R(x_n, y_n),$$

are the real roots of an equation of degree n^2 with coefficients which are rational in terms of the real and imaginary parts of the coefficients in (1) *; the remaining roots of the equation being*

$$R\left(\frac{x_\alpha + x_\beta}{2} + i\frac{y_\alpha - y_\beta}{2}, \quad \frac{y_\alpha + y_\beta}{2} + i\frac{x_\alpha - x_\beta}{2}\right), \quad (\alpha \neq \beta).$$

This result may easily be extended to functions of any number of roots $R(x_1, y_1, x_2, y_2, \cdots)$, and Theorem XIV may be extended to any system of simultaneous equations.

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ALTERNATING CURRENT PHENOMENA.

Alternating Current Phenomena. By C. P. STEINMETZ. New York, Office of the Electrical World. Third Edition, 1900. Pp. xx + 525.

To electrical engineers Mr. Steinmetz's book is immediately conspicuous by reason of two distinguishing characteristics :