

the transcendentals corresponding to  $\pi$  and  $e$  in the  $n$ th process are algebraic numbers borrowed from the  $(n-1)$ th and  $(n+1)$ th processes respectively. The totality of numbers may thus be conceived as arranged in an infinite number of strata, the ordinary algebraic numbers forming a single stratum.

With the extension of the differentiation process, and the stratification of the number body, all the results of the ordinary calculus, for example Taylor's theorem, can be expressed in terms of any two consecutive processes.

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THE VALUE OF  $\int_0^{\pi/2} (\log 2 \cos \varphi)^m \varphi^n d\varphi$ .

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THE integral in question, in which  $m$  is any positive integer, and  $n$  is any even positive integer or zero, is given in effect, for the cases  $m=1, n=0$ , in Williamson's *Integral Calculus*, §118 of the second edition, being taken from a paper by H. G. (presumably Harvey Goodwin, Bishop of Carlisle) in volume 3 of the *Quarterly Journal of Mathematics*. Further it is given in effect, for the case  $m=2, n=0$ , in Wolstenholme's *Problems*, p. 332 of the second edition. It seems worth while to show how it can be expressed in general, in terms of the constants  $s_r = \sum_1^{\infty} 1/n^r$ , which may be regarded as known.

We know that when  $p$  and  $q$  are real

$$1 + \frac{p \cdot q}{1 \cdot 1} + \frac{p(p-1)q(q-1)}{1 \cdot 2 \cdot 1 \cdot 2} + \dots = \frac{\Gamma(1+p+q)}{\Gamma(1+p)\Gamma(1+q)},$$

when  $p+q > -1$  (Forsyth, *Differential equations*, p. 197). But the left member is the constant term in the product of the series

$$(1+x)^p = 1 + px + \dots$$

and

$$(1+1/x)^q = 1 + q/x + \dots$$