

group of isomorphisms of  $C$  is cyclic only when  $\alpha_0 = 0$  or 1 and just one of the other exponents differs from 0, or when  $\alpha_0 = 1$  or 2 and all the other exponents are 0.\*

CORNELL UNIVERSITY,  
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### BESSEL FUNCTIONS.

*Einleitung in die Theorie der Bessel'schen Funktionen.* By PROFESSOR J. H. GRAF und DR. E. GUBLER. Zweites Heft: *Funktionen zweiter Art.* Bern, Wyss and Co., 1900.

THE first part of this work appeared in 1898 and was reviewed in the BULLETIN, February, 1899, pp. 253-8. The general arrangement of the second part is similar to that of the first, the authors again emphasizing the fact that the work is done in the spirit of Schläfli's lectures, the manuscripts of which were in their hands, though many problems are extended and modernized. This fact explains the absence of many important phases of the theory of the Bessel functions which one might expect in a symmetric treatise. Moreover, the authors have been rather overgenerous in their references to papers originating at Bern, omitting others which contained proofs of fundamental theorems prior to their discovery by the Bern school, although probably no plagiarism could be charged. Several fundamental theorems by American authors have received no recognition in the book.

Here, as in Volume I, the loop integral is the principal factor in the investigation, and next in importance is the expansion in series. The differential equation is less frequently used. The procedure is rather original, and frequently markedly different proofs for well-known theorems are given, which in some instances have led to detection of error in papers already published.

The only attempt at a concrete illustration or application is the expansion of a few functions in terms of Bessel functions, though the relations which exist between these functions and others are quite fully brought out.

The second part begins with the expansion of  $\frac{1}{x-y}$  in terms of Bessel functions, the result being

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\* Gauss, Disquisitiones Arithmeticae, 1801, Art. 92.