NOTE ON HAMILTON'S DETERMINATION OF IRRATIONAL NUMBERS.

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THE purpose of this note is to call attention to Hamilton's use of the partition (Schnitt) in his definition of certain irrational numbers.*

Though Hamilton's method of discussing the irrational number is open to serious criticism, it has some interest as affording an approximation to the rigorous theory afterward developed by Dedekind †, and also as presenting another example of the natural character of Dedekind's definition of the irrational number. ‡

For Hamilton time rather than space furnished the rudiment to which his intuition appealed for heuristic purposes and for his conception of unity. Starting with this conception he gives a careful exposition of the operations on positive and negative integers which do not carry one outside the system. A similar discussion of the rational number follows. From the analogy furnished by the laws of operations on integral and rational numbers, Hamilton writes expressions by which he defines the fundamental operations on ratios of any two intervals (steps) of time, and assumes that to any such ratio corresponds a number which he calls an "algebraic number." This he does, even though he has considered the existence of no numbers save integers and fractions In proceeding to develop his theory of irrational number Hamilton too often makes use of these "algebraic numbers," of whose existence and properties he has proved nothing. Throughout his work, however, fractions could be substituted for "algebraic numbers" and such a change would go far toward making his work rigorous In the following outline of Hamilton's work, it is assumed unless the

* "Theory of conjugate functions, or algebraic couples; with a preliminary and elementary essay on algebra as the science of pure time," *Transactions of the Royal Irish Academy*, vol. 17 (1837), p. 293. This essay was read in 1833.

† Stetigkeit und irrationale Zahlen, 1872

‡ In commenting on Tannery's development of the partition (Introduction à la théorie des fonctions d'une variable, 1886) from a remark made by Bertrand (Traité d arithmétique, p. 203, 1849) Dedekind says, (Was sind und was sollen die Zahlen, 1893, p. xiii): "Diese Uebereinstimmung scheint mir ein erfreulicher Beweis dafür zu sein, dass meine Auffassung der Natur der Sache entspricht."