POSSIBLE TRIPLY ASYMPTOTIC SYSTEMS OF SURFACES.

BY DR. L. P. EISENHART.

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IN my note, entitled "A demonstration of the impossibility of a triply asymptotic system of surfaces,"* which was read before the AMERICAN MATHEMATICAL SOCIETY at its December meeting, I failed to take account of an exception which presents itself in the discussion. It was found that in order that there exist a triply asymptotic system of surfaces, it is necessary and sufficient that the double system of asymptotic lines of each surface shall at the same time be geodesic lines on the surfaces. This double condition can be satisfied only when the asymptotic lines in both systems are rectilinear generators of the surface. The quadric surfaces are known to possess this peculiar property, and for the hyperboloid of one sheet and the hyperbolic paraboloid these generatrices are real. Hence instead of the general negation previously given we have the qualified one:

The only triple systems of surfaces cutting mutually in the real asymptotic lines of these surfaces are composed of properly associated families of hyperboloids of one sheet and hyperbolic paraboloids.

One such system can be gotten as follows:

As, in the previous note, we consider space referred to any system of curvilinear coördinates ρ_1, ρ_2, ρ_3 and let the cartesian coördinates x, y, z of a point with respect to fixed rectangular axes be given in terms of ρ_1, ρ_2, ρ_3 by the equations

$$(1) \quad x = f(\rho_1, \rho_2, \rho_3), \quad y = \varphi(\rho_1, \rho_2, \rho_3), \quad z = \psi(\rho_1, \rho_2, \rho_3).$$

We have remarked that the coefficients of the system

$$(2) \quad \begin{aligned} \frac{\partial^2 \theta}{\partial \rho_1^2} &= a_{11} \frac{\partial \theta}{\partial \rho_1} + a_{12} \frac{\partial \theta}{\partial \rho_2} + a_{13} \frac{\partial \theta}{\partial \rho_3}, \\ \frac{\partial^2 \theta}{\partial \rho_2^2} &= a_{21} \frac{\partial \theta}{\partial \rho_1} + a_{22} \frac{\partial \theta}{\partial \rho_2} + a_{23} \frac{\partial \theta}{\partial \rho_3}, \\ \frac{\partial^2 \theta}{\partial \rho_3^2} &= a_{31} \frac{\partial \theta}{\partial \rho_1} + a_{32} \frac{\partial \theta}{\partial \rho_2} + a_{33} \frac{\partial \theta}{\partial \rho_3}, \end{aligned}$$

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