INDIRECT CIRCULAR TRANSFORMATIONS AND MIXED GROUPS.

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1. IN a paper* entitled "Continuous groups of circular transformations," the writer gave a list of the continuous groups of circular transformations in the plane and their chief properties. That paper treated only of direct transformations. The present paper is supplementary to the former, and deals with indirect circular transformations and the mixed groups obtained by combining these with the direct transformations.

I. PROPERTIES OF INDIRECT CIRCULAR TRANSFORMATIONS.

2. Certain fundamental properties of indirect circular transformations were developed by Fricke and published † in 1890. I have seen no paper of more recent date dealing with the subject. Let T be the symbol of a direct, and $\overline{T}$ the symbol of an indirect, circular transformation. Fricke's results, which form the starting point of this paper, may be stated as follows :

The second power of $\overline{T}$ is a direct circular transformation, which is either hyperbolic, parabolic or elliptic. $\overline{T}$ leaves invariant two real points, one real point, or no real points, according as its second power is hyperbolic, parabolic, or elliptic. There are three varieties of indirect transformations, viz., the hyperbolic, parabolic, and elliptic, designated by $\overline{hT}$, $\overline{pT}$ and $\overline{eT}$, respectively, distinguished according to the character of their second powers and hence also according to the number of their invariant points. $\overline{hT}$ and $\overline{hT}^2$ leave invariant the same pair of invariant points A and A'; $\overline{pT}$ and $\overline{pT}^2$ have the same invariant point A ; $\overline{eT}$ has no invariant point, but interchanges the pair of points which $\overline{eT}^2$ leaves invariant. An indirect transformation of period two is an inversion of the plane with respect to a real or pure imaginary circle. Other properties of $\overline{T}$ will now be developed.

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* BULLETIN (2), vol. 4, pp. 107-131 (Dec., 1897).

† Klein-Fricke's Modulfunctionen, vol. 1, pp. 196-207.