DINI'S METHOD OF SHOWING THE CONVERGENCE OF FOURIER'S SERIES AND OF OTHER ALLIED DEVELOPMENTS.

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IF it is required to show that the arbitrary function $f(x)$ of the real variable x may be developed into a Fourier series for all values of x lying between $-\pi$ and π ($-\pi$ and π at most excluded) then, remembering that for any particular value of x, such as $x = a$, the sum of the first $n + 1$ terms of the series is

$$(1) \qquad \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) \frac{\sin \frac{2n+1}{2}(x-a)}{\sin \frac{1}{2}(x-a)} dx,$$

it is easily shown that we need merely examine the limits approached by the integrals

$$(2) \qquad \int_{0}^{d} f(x) \frac{\sin \frac{2n+1}{2}(x-a)}{\sin \frac{1}{2}(x-a)} dx,$$

$$(3) \qquad \int_{c}^{d} f(x) \frac{\sin \frac{2n+1}{2}(x-a)}{\sin \frac{1}{2}(x-a)} dx,$$

as n increases indefinitely, c and d being any numbers such that $0 < c < d \gtreqless \frac{1}{2}\pi$. In case these limits exist and have certain simple properties it becomes evident that the given series will be convergent for any value $x = a$ which lies between $-\pi$ and π, and will have as its sum either $f(a)$ or

$$\frac{f(a+0) + f(a-0)}{2},$$

while at either of the points $x = -\pi$ or $x = \pi$ the sum will be

$$\frac{f(-\pi+0) + f(\pi-0)}{2}$$