

A DEMONSTRATION OF THE IMPOSSIBILITY OF  
A TRIPLY ASYMPTOTIC SYSTEM OF  
SURFACES.

BY DR. L. P. EISENHART.

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Consider in space any system whatever of curvilinear coordinates,  $\rho_1, \rho_2, \rho_3$  and let the Cartesian coordinates  $x, y, z$  of a point with respect to fixed rectangular axes be given in terms of the preceding by the equations

$$(1) \quad x = f(\rho_1, \rho_2, \rho_3), \quad y = \varphi(\rho_1, \rho_2, \rho_3), \quad z = \psi(\rho_1, \rho_2, \rho_3).$$

It is evident that the coefficients of the system of equations

$$(2) \quad \begin{aligned} \frac{\partial^2 \theta}{\partial \rho_1^2} &= a_{11} \frac{\partial \theta}{\partial \rho_1} + a_{12} \frac{\partial \theta}{\partial \rho_2} + a_{13} \frac{\partial \theta}{\partial \rho_3}, \\ \frac{\partial^2 \theta}{\partial \rho_2^2} &= a_{21} \frac{\partial \theta}{\partial \rho_1} + a_{22} \frac{\partial \theta}{\partial \rho_2} + a_{23} \frac{\partial \theta}{\partial \rho_3}, \\ \frac{\partial^2 \theta}{\partial \rho_3^2} &= a_{31} \frac{\partial \theta}{\partial \rho_1} + a_{32} \frac{\partial \theta}{\partial \rho_2} + a_{33} \frac{\partial \theta}{\partial \rho_3} \end{aligned}$$

can be so determined that it will admit  $x, y, z$ , as particular simultaneous solutions.

Any point  $(x, y, z)$  may be looked upon as the intersection of three surfaces, one of each of the families for which  $\rho_1, \rho_2, \rho_3$  are the respective parameters.

Consider a surface of each family and the point defined by them. From the form of equations (2)\* it is evident that the necessary and sufficient condition that the surfaces  $\rho_1 = \text{const.}$  and  $\rho_2 = \text{const.}$  cut out upon the surface  $\rho_3 = \text{const.}$  the asymptotic lines through the given point, is expressed by

$$a_{13} = a_{23} = 0.$$

Since similar results are obtained for the other two surfaces through the point, and in turn for every point and the surfaces determining it, it follows that the necessary and sufficient condition that the surfaces of a triple system cut one another along asymptotic lines is that the coefficients of the equations (1) satisfy the condition

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\* Bianchi, Lezioni, . 109.