ON SINGULAR TRANSFORMATIONS IN REAL PROJECTIVE GROUPS.

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A SINGULAR transformation in a continuous group has been defined as one that can not be generated from an infinitesimal transformation of the group. There have recently appeared in this country several papers* dealing with the question of singular transformations in continuous groups, and in particular in the subgroups of the projective group. The authors of all these papers use the same method, viz., Lie's theory of continuous groups, and consider the variables and parameters to be complex numbers.

In this paper it is proposed to treat those transformations in real projective groups which can not be generated from the real infinitesimal transformations of these groups. I shall make use of a radically new method of treatment, in fact an independent theory of these groups. In what follows the discussion will be limited to real projective transformations in one and two dimensions. The method admits of ready application to three and higher dimensions.

§ 1. *Real Transformations in one Dimension.*

Every projective transformation in one dimension

$$T: \quad x_1 = \frac{ax+b}{cx+d}, \tag{1}$$

can be reduced to one or other of the normal forms

$$(a)\quad \frac{x_1 - A'}{x_1 - A} = k\frac{x - A'}{x - A}, \quad (b)\quad \frac{1}{x_1 - A} = \frac{1}{x - A} + \alpha. \tag{2}$$

In the first form k is the cross ratio of the two invariant points A, A' and the pair of corresponding points x, x_1. For any transformation of this form the cross ratio k is the same for all pairs of corresponding points. The invariant points A, A' and the cross ratio k are the geometric constants of the transformation.

* Rettger, *Amer. Jour. of Math.*, vol. 22, p. 60; *Proc. Amer. Acad.*, vol. 33, p. 493; Williams, *Proc. Amer. Acad.*, vol. 35, p. 97; Taber, BULLETIN, vol. 6, p. 199, and several other papers by the same author. For references to these, see footnote on page 61 of Rettger's paper in *Amer. Jour. of Math.*, vol. 22.