

In Professor Davis's second paper it is pointed out that the 48 operations which bring the cube into coincidence with itself are operations which bring any one of the eight triangles into which a face is divided by its lines of symmetry into coincidence with the 48 like triangles of the surface of the cube. Any of the subgroups of operations brings this same triangle into coincidence with merely a part of the 48 triangles. A cube with these triangles all of one color and the rest of the cube another color is an object which, taking account of color, comes into coincidence with itself only under the operations of the subgroup. Similar methods can be used for the representation of all regular groups and their subgroups.

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## ON THE HISTORY OF THE EXTENSIONS OF THE CALCULUS.

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THE results attained by means of the infinitesimal calculus naturally evoked similar attempts in other directions. We may distinguish between two kinds of new theories, differing from each other mainly in their origin. The one is the natural outcome of an ever recurring need of solving practical problems or of giving existing theories a broader basis; the other owes its origin to purely abstract speculation. The former is not the property of any one mathematician; its nomenclature is not arbitrarily chosen, and its general introduction is only a question of time. This is clear from the history of all those branches of mathematics that have come into general use.

The case is somewhat different with the purely speculative theories. But though these may never succeed in demonstrating their practical utility, yet they are of value to mathematical science since they throw new light on existing methods and on their usefulness. For our purpose, however, a summary outline of these theories will suffice.

The present abstract is confined to those theories that are in close relation to the infinitesimal calculus and the theory of functions, and excludes, for instance, all geometrical methods and what are called "principles" or methods of demonstration.