ON THE GEOMETRY OF THE CIRCLE.

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LET $x_1, x_2, \cdots, x_5$ be any five numbers which satisfy the homogeneous quadratic identity $\omega(x) \equiv 0$; these numbers may be taken as the homogeneous coördinates of the circle.

The linear geometry of the circle can be readily interpreted from the paper of the author * on Dupin's cyclides; and the corresponding theorems for the quadratic configurations from another paper.† In the latter, one of the numbers x_6 was given a restricted interpretation, that of representing all the points of space; and the resulting theorems all referred to cyclides. A set of similar theorems exists for the bicircular quartic curves when, in the geometry of the circle, one coördinate equated to zero represents the points of a plane. All the known theorems regarding these curves, as given by Casey,‡ Darboux,§ and Loria,‖ can be very easily derived, and a number of new ones which are not contained in these memoirs. Another specialization is that obtained by taking the lines of a plane as one of the fundamental complexes. This case has not been systematically treated.

Now suppose that any five complexes mutually in involution be taken as fundamental complexes. In general, none of these complexes is orthogonal, and no circle belongs to them all. The quadratic identity now becomes

$$\omega(x) \equiv \sum_{\tau=1}^{5} x_\tau^2 = 0.$$

These coördinates now represent circles, and not points, as in the memoirs quoted. This theorem results immediately: The curve of singularities of a general quadratic complex is

* V. Snyder, "On the determination of nodes in Dupin's cyclides," *Ann. of Math.*, vol. 11, p. 137.

† V. Snyder, "Geometry of some differential expressions," BULLETIN, vol. 4, p. 144.

‡ J. Casey, "On the bicircular quartics," *Trans. R. Irish Acad.*, vol. 24, p. 359.

§ G. Darboux, "Sur une classe remarquable de courbes et de surfaces," Paris, 1873.

‖ G. Loria, "Sur la géométrie analytique du cercle," *Quar. Jour., of Math.*, vol. 22, p. 44.