ON THE SINGULAR TRANSFORMATIONS OF GROUPS GENERATED BY INFINITESIMAL TRANSFORMATIONS.

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BY means of r independent infinitesimal transformations

$$X_j = \sum_1^n {}_i\xi_{ji}(x_1, x_2, \cdots, x_n) \frac{\partial}{\partial x_i} \qquad (j = 1, 2, \cdots, r)$$

we may construct a family of transformations

$$(1) \qquad x_i' = x_i + \sum_1^r {}_j a_j X_j x_i + \tfrac{1}{2} \sum_1^r {}_j \sum_1^r {}_k a_j a_k X_j X_k x_i + \cdots$$
$$\equiv f_i(x_1, \cdots, x_n, a_1, \cdots, a_r) \qquad (i = 1, 2, \cdots, n)$$

with r essential parameters $a_1, a_2, \cdots, a_r$. The transformations defined by these equations, for assigned values of the a's, may be denoted by T_a. Each transformation of this family is paired with its inverse.

For finite values of the parameters a, the transformation T_a (provided it is not illusory) belongs to a one parameter group generated by the infinitesimal transformation

$$a_1 X_1 + \cdots + a_r X_r.$$

As the a's approach certain limiting values, one or more of which is infinite, T_a may have a definite finite transformation T as a limit. The transformation T may be regarded as a transformation of the family, and, if equivalent to a transformation T_b with finite parameters, can be generated by an infinitesimal transformation of the family (namely, $b_1 X_1 + \cdots + b_r X_r$), but not otherwise.*

Let it be assumed that

$$X_j X_k - X_k X_j = \sum_1^r {}_s c_{jks} X_s \qquad (j,k = 1, 2, \cdots, r),$$

* Thus, if the transformation T_a, for one or more of the a's infinite, is finite and definite, but is not equivalent to a transformation of the family with finite parameters, the transformation T_a cannot be generated by an infinitesimal transformation of the family. To this extent Lie's theorem on p. 65 of the Transformationsgruppen, vol. 1, requires modification.