THE THEORY OF FUNCTIONS.

Introduction to the Theory of Analytic Functions. By J. HARKNESS and F. MORLEY. London, Macmillan and Co., 1898. 8vo, xv + 336 pp.

THE authors, already well known through their Treatise on the theory of functions, have laid the mathematical public under a new obligation by their "Introduction to the theory of analytic functions." The name which the authors have chosen for their new book gives only an inadequate idea of its scope and object; on the one hand it is not an introduction in the sense of a *first* introduction to the theory of functions; on the other hand it is much more than an introduction; it may be shortly described as a very complete *treatise on Weierstrass's general theory of functions* with applications to elliptic and algebraic functions (Chapters VI., XIX., XXI.), preceded by an introduction devoted to the number concept and the geometric interpretation of complex quantities (Chapters I.–V.), and followed by a short account of some of the leading ideas of Riemann and Cauchy (Chapters XX., XXII.).

I. *The Introductory and Concluding Chapters.*

The book begins with a sketch of the system of real numbers considered as *ordinal numbers* (Chapter I.). A row of objects is considered with regard to their order, say from left to right. To count the objects means to label them, not primarily to say how many there are; and the integers are, primarily, mere labels. The object after which we begin to count, is labelled 0; if there are also objects to the left of it, they are labelled -1, -2, etc. Fractions are introduced by a process of relabelling: Pick out of the original row the objects p, $2p$, $3p \cdots$, and relabel them $1, 2, 3, \cdots$; if μ is not divisible by p, the object originally labelled μ is relabelled $\frac{\mu}{p}$. At the same time rules are given to decide whether two rational numbers are equal, and if not, which is the greater. A different kind of relabelling, viz., a change of origin, is used to define addition without reference to quantity. Also special irrationals like $\sqrt{2}$ can be introduced by a similar process of relabelling; but for the general definition of irrationals Dedekind's idea of a cut in