ON ELLIPTIC FUNCTIONS.

BY PROFESSOR JAMES PIERPONT.

How simply and naturally to present the theory of elliptic functions is an embarrassing question. Since the functions of Jacobi and Weierstrass are both used by eminent mathematicians it is necessary to treat them both. This is usually done by giving one of the two classes the preference and deducing the properties of the other as corollaries. Thus in the works of Thomae and Weber the functions of Jacobi are given the preference; in the works of Halphen, Jordan, and Tannery and Molk it is the functions of Weierstrass.

I do not believe this course tends to show to best advantage the peculiar nature of each class of functions nor to show up their similarities and differences.

In my opinion the true foundation of Jacobi's theory is the *thetas*, of Weierstrass's the *sigmas*. To deduce the properties of the functions of Jacobi as Jordan has done in the new edition of his excellent Cours seems to me to rob the reader of half the beauties of this theory. The same is true of the functions of Weierstrass when deduced from those of Jacobi.

Another feature we should look for in a satisfactory presentation of the theory is one too often overlooked. We should demand that, since the sigmas and thetas are the fundamental elements of our theory, these should not be brought on to the scene like a kind of *deus ex machina* but that they appear as necessary elements of our reasoning. The introduction of the sigmas is made easy and natural by virtue of Weierstrass's factor theorem. To introduce the thetas naturally is less easy. Weber has shown with great success how the properties of the functions of Jacobi may be studied by means of the T functions, that is one valued transcendental functions which satisfy the relations

$$\begin{aligned} T(u + 2\omega_1) &= e^{-\pi i[2a_1(u+\omega_1)+b_1]}\,T(u) \\ T(u + 2\omega_2) &= e^{-\pi i[2a_2(u+\omega_2)+b_2]}\,T(u) \end{aligned} \tag{1}$$

where
$$R\left(\frac{\omega_2}{\omega_1}\right) > 0 \tag{2}$$

and
$$2a_2\omega_1 - 2a_1\omega_2 = m, \quad \text{an integer.} \tag{3}$$