# THE GEOMETRY OF MOVEMENT.

*Geometrie der Bewegung in synthetischer Darstellung.* Von Dr. ARTHUR SCHOENFLIES. Leipzig, B. G. Teubner, 1886. 8vo, pp. vi + 194.

*La Géométrie du Mouvement. Exposé synthétique.* Translated by CH. SPECKEL, Capitaine du Génie. Paris, Gauthier-Villars, 1893. 8vo, pp. vii + 292.

PERHAPS apology is needed for noticing a book no longer in its infancy. But we feel that "better late than never" applies to acquaintance with a work which contains so much matter which was new at the time of writing and is not yet accessible in English, nor (we believe) well known.

The main idea of the book is to consider a body in two or more positions relatively to another body, and thence as a limit case to discuss the instantaneous motion. Full advantage is taken of the duality arising from viewing things from the standpoint of the one body or the other. That we have found the book difficult is probably due to our early training; but a few more figures and a few more details would have been welcome.

The French translation is very reliable, and its value is increased by a good elementary account of complexes and congruences of lines (pp. 219–291), by G. Fouret.

Our intention is, not to discuss the information in the book, but to select a few of the more salient theorems (omitting such as are presumably familiar). When in this string of enunciations there seems occasion to interpolate a remark, square brackets are used.

## §1. *Motion of a Plane in a Plane.*

(1) Let $\sigma_0$, $\sigma_1$, $\sigma_2$ be three positions of a plane $\sigma$, relative to a plane $\sigma'$. Let $A_0$, $A_1$, $A_2$ be the three positions of a point $A$ of $\sigma$, $A'$ the point of $\sigma'$ equidistant from $A_0$, $A_1$, $A_2$. The correspondence of the points $A$ and $A'$ is quadratic; to a line in the one plane corresponds a conic in the other, passing through the three centres of rotation in that other.

(2) All the lines of $\sigma$ which for the three positions meet in a point of $\sigma'$ form a pencil in $\sigma$. All the points of $\sigma$ which lie on a line of $\sigma'$ lie on the circumcircle of the centres of rotation in $\sigma$.

This circle is called proleptically the "circle of inflexions."

[Two questions arise here which are not considered in the book: What is the vertex of the pencil in $\sigma$, and what is