PICARD'S ALGEBRAIC FUNCTIONS OF TWO VARIABLES.

Théorie des Fonctions algébriques de deux Variables indépendantes. Par ÉMILE PICARD, Membre de l'Institut, Professeur à l'Université de Paris, et GEORGES SIMART, Capitaine de Frégate, Répétiteur à l'École Polytechnique. Tome I, Paris, Gauthier-Villars et Fils, 1897. 8vo, vi+246 pp.

THE theory of functions of a single complex variable, the growth of which has been one of the striking features in the history of pure mathematics during this century, can, as it is well known, be developed from at least two tolerably distinct points of view. Cauchy and Riemann made extensive use both of methods of proof—such as integration—which may be conveniently called transcendental, and of geometrical reasoning; by such means they established not only results of a geometric or transcendental character but also others which, when once obtained, were naturally expressible in terms of pure algebra. Weierstrass and his school on the other hand have made scarcely any use of geometry, even for purposes of illustration, and have developed the subject from an almost purely algebraical standpoint, building it up systematically and logically from the most elementary notions. The former method has the interest which always attaches to investigations which connect two or more different branches of mathematics and use the methods of the one to solve problems in the other; and the geometric interpretations which continually present themselves are with most minds a valuable aid towards the clear comprehension of the theory. Partly owing to these reasons, partly owing to the well known difficulty of access to Weierstrass's ideas, most systematic treatises on the theory of functions which were published up to a few years ago expounded chiefly the ideas of Cauchy, Riemann and their followers. But, as the Weierstrassian methods have become more widely known, their severe simplicity, their unity, and their rigor have made many converts; and Weierstrass's dictum, that the theory of functions "must be built up on the basis of algebraic truths, and that it is not therefore the right way if conversely the 'transcendental' is employed to establish simple and fundamental algebraical propositions,"* is every year finding more general accept-

* Letter to Professor Schwarz, of October 3, 1875, Mathematische Werke, II, p. 235.