SURFACES OF REVOLUTION IN THE THEORY OF LAMÉ'S PRODUCTS.

BY DR. F. H. SAFFORD.

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THE present paper is a review of an article by Haentzschel * in which he criticises certain results obtained by Wangerin.† The problem treated by Wangerin is to obtain the most general orthogonal surfaces of revolution, such that, if Laplace's equation be written in coördinates corresponding to these surfaces, a solution may be obtained in the form of a Lamé's product with an extraneous factor, *i. e.*,

$$V = \lambda . R . R_1 . \theta. \tag{1}$$

R, R_1, θ are functions respectively of the parameters of the two families of surfaces and the meridian planes, while λ may contain all three parameters. Wangerin shows that λ is $\frac{1}{\sqrt{r}}$, where r is the distance from the axis of revolution to the intersection of the three surfaces. His principal result is that the meridian curves are of the fourth degree and are cyclic curves, while the surfaces are of the same degree. Haentzschel states that the most general surfaces are of the thirty-second degree, the meridian curves being of the sixteenth degree.

Both writers give the following equations :

$$\frac{F'(t + iu) \cdot F_1'(t - iu)}{[F(t + iu) - F_1(t - iu)]^2} = H(t) + H_1(u), \tag{2}$$

$$x + ir = F(t + iu), \quad x - ir = F_1(t - iu) \quad (r = \sqrt{y^2 + z^2}). \tag{3}$$

By (2) two conjugate imaginary functions F and F_1 are to be determined such that the first member shall be the sum of two functions, one of t alone, the other of u alone. From (3) the two families of meridian curves are to be obtained by elimination of t and u respectively.

After differentiating (2) successively with respect to t and u, the result is

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* Reduction der Potentialgleichung, E. Haentzschel, Berlin, 1893.

† *Berliner Monatsberichte*, Feb., 1878.