

This theorem can also be immediately applied to Bessel's functions whose order is not zero. Let $F_n^r(x)$ be any real solution of Bessel's equation

$$(4) \quad \frac{d^2y}{dx^2} + \frac{1}{x} \frac{dy}{dx} + \left(1 - \frac{n^2}{x^2}\right)y = 0.$$

Using polar coördinates r, ϑ we have as a solution of (3) when n is real

$$u = \cos n\vartheta \cdot F_n^r(r),$$

when n is pure imaginary

$$u = e^{in\vartheta} \cdot F_n^r(r).$$

Applying the theorem just quoted to these solutions we get the theorems:

If $n^2 \leq 1$, $F_n(x)$ vanishes at least once in any interval of length $2c = 4.810 \dots$ which does not include the origin.

If $n > 1$, $F_n(x)$ vanishes at least once in any interval of length $2c$

throughout which $|x| > c \left[\csc \frac{\pi}{2n} - 1 \right]$.

As a special application I note that we thus get an upper limit for the value of the smallest root of $F_n(x)$ and thus in particular of $J_n(x)$.

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A GENERALIZATION OF APPELL'S FACTORIAL FUNCTIONS.

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LET $F(s, z) = 0$

be an algebraic equation defining s as function of z . Let R , the corresponding Riemann's surface, be of class p . By a system of crosscuts $a_1, \dots, a_p; b_1, \dots, b_p; c_1, \dots, c_p$ the $(2p + 1)$ -ply connected surface R is changed into a simply connected surface R_{abc} .