ASYMPTOTIC LINES ON RULED SURFACES HAVING TWO RECTILINEAR DIRECTRICES.

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LET l be a generator of a non-developable ruled surface S, and let π_s be the tangent plane to S at a point P on l. As P moves along l, π_s will turn about l in such a way that the range (P) and the axial pencil (π_s) are homographic (Chasles's correlation).

Again, let l be a line of a linear complex C, and let π_c be the polar plane in C of a point P on l. As P moves along l, π_c will turn about l in such a way that the range (P) and the axial pencil (π_c) are homographic (normal correlation).

The two axial pencils (π_s) and (π_c), being homographic with a common range (P), are projective with each other when l belongs to S and to C. These two projective pencils have two self-corresponding planes, π_1 and π_2, such that the point of tangency and pole coincide ; let the corresponding points be P_1 and P_2.

If all the generators l of S belong to C, there will be two points on each, such that the tangent plane and the polar plane coincide. The locus of these points will be a curve traced on the surface, called the *complex curve.*

Let P' be a point on the curve contiguous to P, then PP' is a line of C, hence all the tangents to the curve belong to the complex C. The complex curve is an asymptotic line on the surface, because all its osculating planes are also tangent planes, a characteristic property of the asymptotic lines. This theorem may be stated as follows: *Every ruled surface contained in a linear complex has an asymptotic line, all of whose tangents belong to the complex.*

When the surface is algebraic, the order of this asymptotic line can be easily determined.* Consider any plane α and let it cut the curve in a point K; the polar plane of K with regard to C, which is also the tangent plane to the surface at K, will pass through A, the pole of the plane α. But every line of this polar plane κ of K is a tangent to the sur-

* Picard, "Mémoire sur une application de la théorie des complexes linéaires à l'étude des surfaces et des courbes gauches." *Annales de l'École Normale*, 1877.