THE LARGEST LINEAR HOMOGENEOUS GROUP WITH AN INVARIANT PFAFFIAN.

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1. IN the December number of the BULLETIN (pp. 120–135) I have shown that the second compound of the general $2m$-ary linear homogeneous group is a linear group in $C_{2m,2} \equiv m(2m-1)$ variables which leaves invariant the Pfaffian

$$F \equiv [1, 2, \cdots, 2m].$$

Denoting the variables as follows:

$$(1) \qquad Y_{ij} \equiv - Y_{ji} \qquad (i, j = 1, \cdots, 2m\,;\ i \neq j),$$

the second compound was proved to contain exactly $(2m)^2$ linearly independent infinitesimal transformations

$$(2) \qquad \sum_{r \neq s,\, t}^{r=1,\cdots,2m} Y_{rt} \frac{\partial f}{\partial Y_{rs}} \delta t. \qquad (t, s = 1, \cdots, 2m).$$

The object of the present note is to prove that the largest linear homogeneous group G in the $m(2m-1)$ variables (1) which leaves invariant the Pfaffian F contains only the $(2m)^2$ linearly independent transformations (2).

2. Let the general infinitesimal transformation of the group G be as follows:

$$(3) \qquad \delta Y_{ij} = \sum_{k \neq l}^{k=1,\cdots,2m} a_{kl}^{ij} Y_{kl} \delta t \qquad (i, j = 1, \cdots, 2m\,;\ i \neq j),$$

where, on account of (1), we may suppose

$$(4) \qquad a_{kl}^{ij} = - a_{lk}^{ij} = + a_{lk}^{ji}.$$

The condition that (3) shall multiply F by a constant $c\delta t$ is as follows:

$$(5) \qquad \sum_{i,j,k,l} \frac{\partial F}{\partial Y_{ij}} a_{kl}^{ij} Y_{kl} = cF.$$

Now