DETERMINANTS OF QUATERNIONS.

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(Abstract, read before the American Mathematical Society at the Meeting of February 25, 1899.)

I ADOPT as the basis of the following discussion the convention that the order of factors in every term of a determinant is the same as the order of columns in the matrix; while the order of ranks in each term fixes the sign of the term, according to the usual rule for determinants of scalars. It follows that, in quaternions, columns and ranks are not interchangeable; that the transposition of ranks causes the same changes of sign as in scalar algebra; and that a determinant which has two equal ranks vanishes; but that the position of a column is, in general, fixed; and that a determinant may have two or more equal columns without vanishing. If, however, a determinant has a scalar column, such column may be displaced with the same changes of sign as in scalar algebra; and if two scalar columns of a matrix are equal, the determinant vanishes.

The usual development of a determinant as a sum of products of constituents from any row into corresponding minors can be employed only for the first and last columns of a determinant of quaternions, or for a scalar column or rank; but a more general formula, preserving the order of factors in each term, may be employed for *any* column or rank. The addition theorem holds for determinants of quaternions; but the multiplication theorem does not hold, since its proof involves the commutative property of multiplication. The latter theorem may be used, however, when one of the factors is a determinant of scalars; a case which may often occur in applications.

A *Repeating Determinant* is defined as a determinant of which every column is either equal or conjugate to an assumed standard column; and a *Resultant* of several quaternions, as a repeating determinant found from those quaternions, and divided by the number of its terms, that is by the factorial of its order. A single quaternion has two resultants, which are equal respectively to the quaternion and to its conjugate; and n quaternions have 2^n resultants, among which we distinguish two *uniform* resultants, of which the columns are all equal, and two *alternating* result-