

period. If for a point there should be a fifth circuit of any period then there will be an infinite number of circuits of the same period. That such points exist for any particular period appears at once from a study of the number of conditions of contact and the number of parameters involved in them. The process here employed is adequate to produce the locus of points which admit circuits of any period, but for periods higher than four the eliminations become exceedingly complicated.

Many interesting phases of this problem appear by making certain transformations of the plane. For instance, a projection of the plane will convert our configuration into the more general one consisting of lines through a point and an equal number of conics through four points, each line tangent to two conics, and each conic touched by two lines. An inversion would convert the lines of the circuit into circles of a coaxial system, leaving the circles of the circuit still circles of a coaxial system. Thus our configuration would come to consist of a given number of circles of one coaxial system and an equal number of a second coaxial system, each circle of either set touching two of the other set, the whole forming a continuous chain. Our locus would then become the locus of one of the four intersection points of the two systems of circles, which moves, the other three remaining fixed, so as always to make such a chain of given period possible.

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RECIPROCAL TRANSFORMATIONS OF PROJECTIVE COÖRDINATES AND THE THEOREMS OF CEVA AND MENELAOS.

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1. Among the great number of correspondences between certain configurations of the plane and space it is interesting and valuable to consider relations of the triangle in connection with certain surfaces. It will be seen that propositions of plane geometry interpreted in Cartesian space lead to geometrical questions of a more general character. In this paper we shall confine ourselves to the theorems of