THE THEOREMS OF OSCILLATION OF STURM AND KLEIN. (THIRD PAPER.)

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THE following pages form a continuation of two papers presented under the same title to the Society and printed on pp. 295–313 and 365–376 of the preceding volume of the BULLETIN. These papers will be referred to as *Th. of Osc. 1* and *2* respectively.

The object of the present paper is to extend the results so far established to some cases in which the coefficients of the differential equation in question are no longer continuous throughout the intervals with which we are concerned. Such extensions are made in §§ 2 and 3 of the present paper but for this purpose it is necessary in § 1 to establish some fundamental theorems concerning linear differential equations of the second order with discontinuous coefficients, results which are perhaps of some interest apart from the special applications here made of them. Before entering on these questions, however, it will be convenient to describe accurately the kind of discontinuities with which we shall deal, and to establish certain general theorems.

All the functions with which we shall have to deal in the present paper are, throughout the interval in which we consider them, single valued *real* functions of one or more *real* variables. Taking first the case of a function of a single variable $f(x)$, we shall consider only the case in which this function has in an interval $a \leqq x \leqq b$ a finite number of points of discontinuity.

The simplest discontinuities from some points of view are the so-called *finite discontinuities** of $f(x)$, *i. e.*, discontinuities $x = c$ for which a positive quantity M can be found such that in the neighborhood of c, $|f(x)| < M$. Going beyond these finite discontinuities we have discontinuities at

* Strictly speaking this includes the case in which the discontinuity is simply due to the fact that the function has not been defined at the point in question; for example e^{-1/x^2} at the origin. We will, however, here once for all make the convention that in such cases we will regard the function as being so defined at the point in question as to preserve the continuity if possible.