representing $-\varphi(x)$ within this circle all of Poincaré's analysis applies without modification. Hence this circle is the true circle of convergence for this series.

Finally, for the case that x_0 is any point of A, Poincaré's reasoning, with the modification just given, still holds, and the theorem is thus established that $\varphi(x)$ is analytic in A, but cannot be continued beyond A.

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SUPPLEMENTARY NOTE ON A SINGLE-VALUED FUNCTION WITH A NATURAL BOUNDARY, WHOSE INVERSE IS ALSO SINGLE-VALUED.

BY PROFESSOR W. F. OSGOOD.

(Read before the American Mathematical Society at its Fifth Summer Meeting, Boston, Mass., August 19, 1898.)

IN the June number of the BULLETIN I gave an example of a single-valued function with a natural boundary, the inverse of which is also single-valued. The function employed was the following:

$$f(z) = z + \frac{z^{a+2}}{(a+1)(a+2)} + \frac{z^{a^2+2}}{(a^2+1)(a^2+2)} + \cdots,$$

where a is a positive integer greater than unity. This function is continuous within and on the boundary of the unit circle, is analytic within this circle, and cannot be continued analytically beyond it.

I am indebted to Professor Hurwitz for an exceedingly simple proof of the principal theorem of my note, namely, that the inverse function is single-valued. The point to be established is that, z, z' being any two distinct points within or on the unit circle,

$$f(z) \neq f(z').$$

This follows at once by the application of a method employed by Professor Fredholm* to show that the inverse of the function

* *Cf.* Verhandlungen des ersten internationalen Mathematiker-Kongresses in Zürich vom 9. bis 11. August 1897; herausgegeben von Dr. Ferdinand Rudio, Professor am eidgenössischen Polytechnikum; Teubner, 1898; p. 109.