INFINITESIMAL TRANSFORMATIONS OF CONCENTRIC CONICS.

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A FAMILY of curves is invariant under the transformations of a continuous group of transformations when the family is invariant under the infinitesimal transformations which generate the group. A family is invariant under an infinitesimal transformation when the differential equation of the family admits of the infinitesimal transformation.

The criterion that a given differential equation of the mth order in x, y

$$\Phi(x, y, y', y'', \cdots, y^{(m)}) = 0, \tag{1}$$

admit of a known infinitesimal point transformation

$$Uf \equiv \xi(x, y)f\frac{\partial f}{\partial x} + \eta(x, y)\frac{\partial f}{\partial y}, \tag{2}$$

is that

$$U^{(m)}\Phi \equiv 0, \tag{3}$$

where

$$U^{(m)}f \equiv \xi\frac{\partial f}{\partial x} + \eta\frac{\partial f}{\partial y} + \eta'\frac{\partial f}{\partial y'} + \cdots + \eta^{(m)}\frac{\partial f}{\partial y^{(m)}},$$

$$\eta' \equiv \frac{d\eta}{dx} - y'\frac{d\xi}{dx}, \cdots, \eta^{(m)} \equiv \frac{d\eta^{(m-1)}}{dx} - y^{(m)}\frac{d\xi}{dx},$$

give the mth extension of the original point transformation, Uf.

Conversely, if the differential equation is given and the infinitesimal transformation unknown, the condition (3) may be turned to account to find the forms of those infinitesimal point transformations of which the given differential equation admits. This converse problem is an integration problem not capable of general solution ; in fact there are differential equations of the mth order which do not admit of infinitesimal point transformations.

If in particular the equation (1) is of the second order and can be put in the form

$$y'' - \omega(x, y, y') = 0, \tag{4}$$