THE STRUCTURE OF THE HYPOABELIAN GROUPS.

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1. THIS paper gives a marked simplification both in the general conceptions and in the detailed developments of the theory of the two hypoabelian groups of Jordan and of the writer's generalization [*] to the Galois field of order 2^n of the first hypoabelian group. It is important, especially for the generalization, to give these groups an abstract definition independent of the theory of " exposants d'échange," by means of which Jordan derived them. The crucial point in the simplified treatment lies in the discovery of the explicit relations

$$\sum_{i,j}^{1\ldots m} a_j^{(i)}\delta_j^{(i)} = m, \quad \sum_{i,j}^{1\ldots m} a_j^{(i)}\delta_j^{(i)} + a_1' + \beta_1' + \gamma_1' + \delta_1' = m,$$

satisfied by the substitutions of the simple sub-groups J and J_1, respectively, but ruling out the remaining substitutions of the total hypoabelian groups G and G_1. We may therefore avoid the dependence made [†] in §§ 274 and 289 upon the last book of the Traité (see §672, page 506).

Basing the investigation upon the groups J and J_1, which are to be proved simple, and not upon G and G_1 as in the earlier treatments, we wholly avoid the delicate analysis and calculations necessary in §§275 and 290. For the first hypoabelian group, the sub-division into cases is diminished one-half. For the second hypoabelian group, decided simplifications may be made in §§284, 286–8. Some errors have been detected ; thus the groups G and G_1 do not have the same order, as stated in Jordan, §279. §291 is wholly wrong.

2. The groups G and G_1 are sub-groups of the *simple* [‡]

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[*] " The first hypoabelian group generalized," *The Quarterly Journal of Mathematics*, 1898.

[†] The indefinite references in Jordan remained an enigma to me until quite recently. Jordan himself could not recall them upon my personal request last year.

[‡] Dickson : "A triply infinite system of simple groups," *The Quarterly Journal*, July, 1897.