That $V(x, y)$ thus defined is harmonic follows at once from (3) since $\frac{\partial \psi}{\partial s}$ is easily seen, either by direct computation or from its value:

$$\frac{\partial}{\partial n}[\log \rho_1 - \log \rho]$$

to be a harmonic function of (x, y).

For the proof of the second part of the theorem formula (2) is particularly adapted. We have here to prove that if (x, y) approaches a point P on the circumference $V(x, y)$ approaches as its limit the value of V_c at P. The idea upon which this proof rests is that when (x, y) is near to P a small arc including P corresponds to a large range of values of ψ and, therefore, when we take the arithmetic mean as indicated in (2) the value of V_c at P will predominate.* The exact proof based upon the idea just stated merely requires the writing down of a few inequalities.

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ON THE POLYNOMIALS OF STIELTJES.

BY PROFESSOR E. B. VAN VLECK.

(Read before the American Mathematical Society at the Meeting of April 30, 1898.)

BY a Stieltjes polynomial will here be understood any polynomial satisfying a regular linear differential equation of the second order

$$\frac{d^2y}{dx^2} + \left(\frac{1-\lambda_1}{x-e_1} + \cdots + \frac{1-\lambda_r}{x-e_r}\right)\frac{dy}{dx} + \frac{\varphi(x) = A_0x^{r-2} + A_1x^{r-3} + \cdots + A_{r-2}}{(x-e_1)\cdots(x-e_r)}y = 0 \tag{I}$$

in which the singular points $e_1, \cdots, e_r, \infty$ are real and in which also r exponent-differences $\lambda_1, \cdots, \lambda_r$ are (algebraically) less than unity. We shall here for the most part confine our

* It will be seen that this idea is similar to that suggested by Schwarz. (Ges. Werke, vol. 2, p. 360. See also Klein-Fricke: Modulfunctionen, vol. 1, p. 512.) We avoid, however, the artificiality of Schwarz's method.