Consider those values of w that yield values of z for which $F(z)$ is defined, and for which then $F(z)$ is a function of w. These values of $F(z)$ do not constitute an analytic function of w; for the domain of values of w consists of two separate continua. Thus the theorem, unrestricted, would be false in this case. *

HARVARD UNIVERSITY,
April, 1898.

NOTE ON POISSON'S INTEGRAL.

BY PROFESSOR MAXIME BÔCHER.

(Read before the American Mathematical Society at the Meeting of April 30, 1898.)

THE following treatment of Poisson's integral in two dimensions seems to the writer to have at least one advantage over the treatments ordinarily given; viz., that it involves no artifice.

Given a function $V(x, y)$ which within and upon the circumference of a certain circle C is a continuous function of (x, y) and within C is harmonic (*i. e.*, has continuous first and second derivatives and satisfies Laplace's equation). By a well-known theorem of Gauss the value of V at the centre (x_0, y_0) of C is the arithmetic mean of its values on the circumference.† That is, if we denote by V_c the values of V on the circumference and by φ the angle at the centre,

$$(1) \qquad V(x_0, y_0) = \frac{1}{2\pi}\int_0^{2\pi} V_c d\varphi.$$

This theorem may be immediately generalized by the method of inversion, if we remember on the one hand that a harmonic function remains harmonic after inversion, and on the other hand that angles are unchanged by inversion and that circles invert into circles. We thus get the theorem:

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* Burkhardt has given simple examples of multiple-valued functions for which the unrestricted theorem is false. See his book: "Einführung in die Theorie der analytischen Functionen einer complexen Veränderlichen," vol. 1, Leipzig, 1897; p. 198.

† An elementary proof of this theorem will be found in a paper by the writer on p. 206 of the BULLETIN for May, 1895.